Egg size and embryo temperature.

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INTRODUCTION

Shanawany (1993) reports that the mass of one-day-old chickens is positively correlated to the mass of its hatching-egg. One-day-old chickens native from motherhens kept at ahemeral cycles (28 h at 14L:14D) are more heavy compared to hemeral cycles (24 h at 14L:8D), thus their eggs are heavier too. Shanawany measured a shorter hatching period for eggs layed at long ahemeral cycles. Explanation?

The delay between first and last pipped eggs is about 36 h in hatcheries although the genetic resemblance of motherbirds and homogeneity of the temperature and relative humidity <u>of the air</u> in the incubator. What can be the reason?

The purpose of this note is to demonstrate the possibilities of pure physical calculations to enlarge the set of explanations for the interpretation of physiological experiments.

EXCESS TEMPERATURE IN HEAT GENERATING BODIES

Many questions of the above type can be approached, sometimes successfully, with 'Physical Transport Phenomena'. An excellent equation to remember and to use is the excess temperature of heat generating bodies:

$$T_{centre} - T_{ambient} = \frac{qR^2}{n\lambda}\left(1 + \frac{2\lambda}{HR}\right) \quad (1)$$

I was not able to trace the inventor of this equation or the person who used it for practical applications. Perhaps it was Pohlhausen or Poggendorf. Most probable he was a geologist, for all famous books concerning heat diffusion are written by geologists: Carslaw and Jaeger, Luikov. Indeed, the centre temperature of the earth is easily calculated with the equation (see appendix).

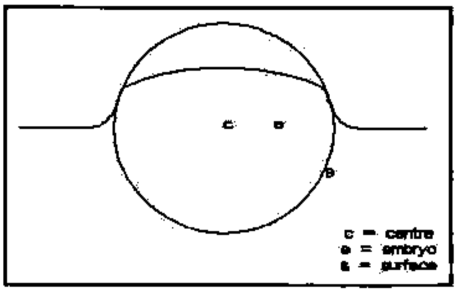

FIG 1 The excess temperature in a heat generating sphere.

Equation 1 holds for all basic geometries if the geometrical factor n is taken as: 2 for wall or plate, 4 for cilinder, 6 for a sphere and 4.4 for a cube. The volumetric heat generation q and thermal conductivity λ must be constant. With excess temperatures over 2 °C this assumption, certainly for the heat generation, is not valid. Equation 1 gives the steady state parabolic temperature distribution in a heat generating body if $-r^2/R^2$ is added to the term between parenthesis.

APPLICATIONS OTHER THEN EMBRYO TEMPERATURE

The equation is used to calculate the density of fission bars in a nuclear pile. The Russians Pomerantsev and Predvoditelev pioneered proposing analytical solutions (see appendix) to problems with non-constant heat sources, see fig 1, and they are honoured by the dimensionless numbers *Po* and *Pd*, like *Nu* (Nusselt).

Gunpowder also releases heat and researchers living in the epoch of Napoleon Bonaparte used scientific modelling with green beans to evaluate the safe radius of powder-barrels. Frank-Kamenetzky and Van Geel did use the inverted equation to calculate the safe radius of a heat generating body in which the heat is produced by a Q10 model.

Fig 2 shows the heat flows for a heat generating body at a constant ambient temperature. De convective heat flow is linear with the temperature of the body while the heatproduction is exponential. As a consequence their are 2 equilibrium points, a stable one at low temperatures and a 'explosive' one at high temperatures. If the temperature of the body exceeds the second equilibrium temperature then the body will desintegrate.

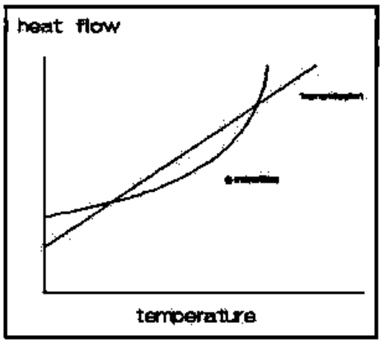

FIG 2 Stable and unstable equilibrium temperature points for heat generating bodies stored at constant ambient temperature.

The safe radius theory was used to prevent haystack fires, storage of fertilizers, seatransport of peroxides and keepability of horticultural produce. Giant-pumpkin growers have a dangerous hobby. Calculate the safe radius yourself if the heat production is 140 W/m^3 at 15 °C.

$$R_s = \frac{\lambda}{H}\left(\sqrt{\left(1 + \frac{10nH^2}{q\lambda}\right)} - 1\right) \qquad (2)$$

The safe radius of a giant-pumpkin is 38 cm when the value of the properties are: thermal conductivity of pumpkin flesh 0.4 W/(m.K), heat transfer coefficient 10 W/(m^2.K). The excess temperature ΔT in a normal sized pumpkin, with radius R = 0.1 m, is:

$$\Delta T = 0.583 * (1 + 0.8) - 1 \text{ °C}$$

jump from surface to ambient
jump from centre to surface

The excess temperature exists of 2 temperature differences, from centre to surface, depending on the heat generation, and from surface to ambient, depending on the convective heat transfer.

EXCESS TEMPERATURE IN HATCHING EGGS

The effect of air velocity on embryonic temperature depends on the approximation of the heat transfer coefficient. For spheres the relation is:

$$Nu = 2 + 1.3Pr^{0.15} + 0.66Re^{0.50}Pr^{0.33} \tag{3}$$

The dimensionless numbers are defined as:

$$Nu = \frac{HD}{\lambda} \quad Pr = \frac{\nu}{a} \quad Re = \frac{vD}{\nu} \tag{4}$$

The heat transfer of eggs varies between H = 10 and 75 $W/(m^2.K)$, see table 1. The value of the properties of air are: thermal conductivity 0.03 W/(m.K), kinematic viscosity 1.5×10^{-5} m^2/s and thermal diffusivity 2.4×10^{-5} m^2/s. The effect of diameter and velocity is pronounced: small eggs have large transfer coefficients, high velocities produce large transfer coefficients.

TABLE 1
Heat transfer coefficient H in $W/(m^2.K)$ of spheres in an air flow calculated with equation 3.

Velocity m/s	Diameter in cm 2	4	6	8
0.5	27.4	18.3	14.6	12.5
1.0	36.7	24.9	20.0	17.1
2.0	49.9	34.3	27.6	23.2
5.0	76.1	52.8	42.7	36.8

Only with heat generation in the egg an excess temperature will develop. To compare the effect of its diameter on the excess temperature we first assume a constant heat source Q, for instance 0.1 W. The volumetric heat generation depends on the heat source and the radius:

$$q = \frac{3Q}{4\pi R^3} \tag{5}$$

Substitution of equation 5 in equation 1 gives the excess temperature of a sphere with a heat source Q:

$$\Delta T = \frac{Q}{8\pi R\lambda} \left(1 + \frac{2\lambda}{HR} \right) \tag{6}$$

The excess temperature of an egg decreases with increasing radius. Small eggs compared to large eggs with the same heat generation have high excess temperatures (table 2). The temperature jump related to the heat generation is smaller than the jump related to the air velocity. The thermal conductivity of the eggs material is corrected for the effect of heat transfer by the blood flow. Without this correction the thermal conductivity is about

0.5 W/(m.K).

TABLE 2
The calculated excess temperature of spheres with a heat generation of Q = 0.1 W at an air velocity of 0.5 m/s and thermal conductivity of the egg λ = 1.23 W/(m.K).

Diameter in cm	Excess temperature centre	embryo	surface
4	1.25	1.21	1.09
5	0.93	0.90	0.80
6	0.71	0.69	0.61

According to air velocity measurements carried out by the Spelderholt in commercial incubators, the minimum and maximum velocity was 0.3 and 2.5 m/s. This spread in air velocity will induce different embryo temperatures in the hatching eggs depending on the actual position in the incubator (table 3). The calculated differences between the excess temperature at various velocities can contribute to the time spread of pipping.

TABLE 3
Excess temperature in a sphere with a diameter of 5 cm, a heat source Q = 0.1 W as a function of the air velocity.

Velocity m/s	Excess temperature in K centre	embryo	surface
0.5	0.93	0.89	0.80
2.5	0.53	0.50	0.40

The physiological effect on the embryo by different excess temperatures of embryos must be measurable although the temperature differences are smaller than 1.0 °C. Growth rate of the embryo, chemical reaction rates in the cells, enzymatic action in organelles are all affected by temperature. The error, induced by experimental method, in a statistical analysis (ANOVA) of experiments with hatching eggs, can be decreased by really using constant air velocity in incubators.

Shanawanys statement about the short hatching time for heaver one-day-old birds can now be judged. Short hatching times are related to higher temperatures of the embryo. The heavier eggs must have as a consequence higher temperatures. The heat production is related to mass of physiological active tissue, for instance 6 W/kg, and has a linear relationship with the volume of the egg. Therefore the volumetric heat production is independent of the egg size: a reasonable quess is q = 1600 W/m^3, equivalent to 0.1 W for an egg with a radius of 5 cm. According to calculation with equation 1 the embryo temperature of heavier eggs is slightly higher compared to lighter ones. A temperature difference of 0.2 °C might be detectable as a change of hatching time.

TABLE 4
Excess temperature in K of different sized hatching eggs with constant volumetric heat generation $q = 1600$ W/m^3, heat transfer $H = 14$ W/(m^2.K) and low thermal conductivity $\lambda = 0.5$ W/(m.K).

Diameter cm	Mass g	Centre	Embryo	Surface
4	34	0.93	0.88	0.73
5	67	1.23	1.15	0.91
6	116	1.55	1.44	1.09

What is the excess temperature in a hatching egg of an ostrich? The apparent diameter of its egg is 13 cm, the mass is 1180 g and the hatching time 40 days at low relative humidity (30 %). The heat generation in the egg is $Q = 3.2$ W when the mass of the embryo is $m = 800$ g (70 % of mass of egg) and the specific heat generation about 4 W/kg. While the relative humidity is very low it is correct to introduce a correction on the heat source of - 0.1 W to account for moisture loss (wish I knew the transpiration coefficient). The excess temperature is about 6 °C if the heat transfer coefficient is taken as $H = 10$ W/(m^2.K). I guess that the best air temperature in an incubator is 33 °C at the end of the hatching.

APPENDIX
CENTRE TEMPERATURE OF EARTH

The radius of the earth is 6000 km. The mean geothermal gradient is 0.03 K/m while the thermal conductivity of the crust is about 2 W/(m.K). The heat transfer through the crust is 2.72 10^{13} W, thus the 'apparent' heat generation of the earth is $q = 3\ 10^{-8}$ W/m3. This heat generation is very low compared to the generation of the pumpkin, but the enormous radius of the earth will boost up the centre temperature. The thermal conductivity of the kernel material is about 50 W/(m.K), comparable to liquid iron. The calculated centre temperature of the earth is, when you disagree measure it yourself:

$$T = \frac{qR^2}{6\lambda} = \frac{3\ X10^{-8} \times 6^2\ X10^{12}}{6\times 50} = 3600\ oC \qquad (7)$$

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with $w/c\gamma$ and adding the value $(w/c\gamma)\tau$ which is the original function of the transform $w/s^2c\gamma$.

Hence the solution of our problem will have the form

$$\theta = \frac{t(r,\tau) - t_0}{t_a - t_0}$$
$$= 1 + \frac{1}{6}\,\mathrm{Po}\left(1 + \frac{2}{\mathrm{Bi}} - \frac{r^2}{R^2}\right)$$
$$- \sum_{n=1}^{\infty}\left(1 + \frac{\mathrm{Po}}{\mu_n^2}\right) A_n \frac{R \sin \mu_n(r/R)}{r\mu_n} \exp[-\mu_n^2\,\mathrm{Fo}], \qquad (8.3.7)$$

where $\mathrm{Po} = wR^2/\lambda(t_a - t_0)$ is the Pomerantsev number, A_n are the initial thermal amplitudes determined by the corresponding relations (see Chapter 6, Eqs. (6.5.28) and (6.5.29)).

In a stationary state we shall have a parabolic law of temperature distribution.

The mean temperature of the sphere required for calculation of the specific heat rate is equal to

$$\bar{\theta} = \frac{\bar{t}(\tau) - t_0}{t_a - t_0}$$
$$= 1 + \frac{1}{15}\,\mathrm{Po}(1 + (5/\mathrm{Bi})) - \sum_{n=1}^{\infty}(1 + (\mathrm{Po}/\mu_n^2))B_n \exp[-\mu_n^2\,\mathrm{Fo}]. \qquad (8.3.8)$$

c. Solution for $w = w_0 e^{-k\tau}$. Applying a similar method of calculation we shall obtain the solution in the form

$$\theta = 1 - \frac{\mathrm{Po}}{\mathrm{Pd}}\left[1 - \frac{R\,\mathrm{Bi}\sin\{(\mathrm{Pd})^{1/2}\,r/R\}}{r[(\mathrm{Bi}-1)\sin(\mathrm{Pd})^{1/2} + (\mathrm{Pd})^{1/2}\cos(\mathrm{Pd})^{1/2}]}\right]\exp(-\mathrm{Pd}\,\mathrm{Fo})$$
$$- \sum_{n=1}^{\infty}\left(1 - \frac{\mathrm{Po}}{\mathrm{Pd} - \mu_n^2}\right) A_n \frac{R\sin\mu_n(r/R)}{r\mu_n}\exp[-\mu_n^2\,\mathrm{Fo}], \qquad (8.3.9)$$

where Pd is the Predvoditelev criterion; in the present case it is equal to $\mathrm{Pd} = (k/a)R^2$, Po is the Pomerantsev criterion, $\mathrm{Po} = w_0R^2/\lambda(t_a - t_0)$.

d. Solution for $w = \dot{w}_0\tau^{n/2}$. The specific strength of the heat source is some power function of time; $w = \dot{w}_0\tau^{n/2}$, where $n = -1, 0, 1, 2, \cdots$. We solve the problem by considering it the boundary condition of the first kind ($\mathrm{Bi} = \infty$), i.e., $t(R,\tau) = t_a$. Using the method given above we obtain the solution in the form

FIG 1 Copy of page 366 from the book of Luikov where the *Po* and *Pd* numbers are for the first time introduced in the book.

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SYMBOLS AND UNITS OF PROPERTIES

symbol	unit	property
$a = 2.4\ 10^{-5}$	m^2/s	thermal diffusivity of air
D	m	diameter of sphere
H	$W/(m^2.K)$	heat transfer coefficient
n	-	geometrical factor
Nu	-	Nusselt number, heat transfer
Pr	-	Prandtl number
q	W/m^3	volumetric heat production
Q	W	heat source
R	m	distance from centre to surface
Rs	m	safe radius
Re	-	Reynolds number, velocity
T	°C	temperature
v	m/s	air velocity
ΔT	K	excess temperature
λ	W/(m.K)	thermal conductivity
$\nu = 1.5\ 10^{-5}$	m^2/s	kinematic viscosity of air

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